§6: Summary

- $f(x)$ is an "antider." of $g(x)$ if

  $f'(x) = g(x)$ $\left(\text{or } \frac{d}{dx}(f(x)) = g(x)\right)$

- From §5.5, if $f'(x)$ is differentiable function on $[a,b]$, then

  $$f(b) - f(a) = \int_a^b f'(x)\,dx \qquad \text{(pg 283)}$$

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$ • $\int \frac{1}{x}\,dx = \ln|x| + C$

- $\int k\,dx = kx + C$ • $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

- $\int (f \pm g)\,dx = \int f\,dx \pm \int g\,dx$

- $\int c\,f(x)\,dx = c \cdot \int f(x)\,dx$

§6.1: Problems (1 to 88, ignore trig.)

19: Find antider. of $g(t) = t^2 + t$.

want to find $G(t)$ such that $G'(t) = g(t)$

$$t^2 + t = \frac{d}{dt}\left(G(t)\right) \qquad \left(G(t) = G_1(t) + G_2(t)\right)$$

$$= \frac{d}{dt}\left(\left(G_1(t) + G_2(t)\right)\right)$$

$$= \frac{d}{dt}\left(G_1(t)\right) + \frac{d}{dt}\left(G_2(t)\right)$$

Want to find G_1, G_2 s.t. $\frac{d}{dt}G_1 = t^2$ and $\frac{d}{dt}G_2 = t$.

$$t^2 = \frac{d}{dt}\left(\frac{1}{3}t^3\right) \Rightarrow G_1(t) = \frac{1}{3}t^3$$

$$t = \frac{d}{dt}\left(\frac{1}{2}t^2\right) \longrightarrow G_2(t) = \frac{1}{2}t^2$$

$$G(t) = G_1(t) + G_2(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2$$

$$\Rightarrow \int (t^2 + t)\,dt = \frac{1}{3}t^3 + \frac{1}{2}t^2 + C$$

22: Find an antider. of $g(t) = t^7 + t^3$.

$$\frac{d}{dt} G(t) = t^7 + t^3$$

$$t^7 = \frac{d}{dt}\left(\frac{1}{8} t^8\right), \quad t^3 = \frac{d}{dt}\left(\frac{1}{4} t^4\right)$$

$$\frac{d}{dt}\left(\frac{1}{8} t^8 + \frac{1}{4} t^4\right) = \frac{d}{dt}\left(\frac{1}{8} t^8\right) + \frac{d}{dt}\left(\frac{1}{4} t^4\right)$$
$$= t^7 + t^3$$

$$\int (t^7 + t^3)\,dt = \frac{1}{8} t^8 + \frac{1}{4} t^4 + C$$

28: Compute $\int (x + x^5 + x^{-5})\,dx$.

$$\int (x + x^5 + x^{-5})\,dx = \int x\,dx + \int x^5\,dx + \int x^{-5}\,dx$$
$$= \left(\frac{x^2}{2} + C_1\right) + \left(\frac{1}{6} x^6 + C_2\right) + \left(-\frac{1}{4} x^{-4} + C_3\right)$$
$$= \frac{x^2}{2} + \frac{1}{6} x^6 - \frac{1}{4} x^{-4} + C \quad (C = C_1 + C_2 + C_3)$$

34: Compute $\int \frac{1}{z^3}\,dz$.

$$\int \frac{1}{z^3}\,dz = \int z^{-3}\,dz = \frac{z^{-3+1}}{-3+1} + C$$

$$= \frac{z^{-2}}{-2} + C$$

36: Compute $\int \left(x^6 - \frac{1}{7x^6}\right)dx$.

$$\int \left(x^6 - \frac{1}{7x^6}\right)dx = \int x^6\,dx + \int \left(-\frac{1}{7x^6}\right)dx$$

$$= \int x^6 dx + \int \left(-\frac{1}{7}\right)x^{-6}\,dx$$

$$= \int x^6 dx - \frac{1}{7}\int x^{-6}\,dx$$

$$= \frac{1}{7}x^7 + C_1 - \frac{1}{7}\left(\frac{x^{-6+1}}{-6+1} + C_2\right)$$

$$= \frac{1}{7}x^7 + C_1 - \frac{1}{7}\left(\frac{x^{-5}}{-5} + C_2\right)$$

$$= \frac{1}{7}x^7 + C_1 - \frac{1}{7}\left(-\frac{1}{5}\right)x^{-5} + \left(-\frac{1}{7}\right)C_2$$

$$= \frac{1}{7}x^7 + C_1 + \frac{1}{35}x^{-5} + C_3 \quad \left(C_3 = -\frac{1}{7}C_2\right)$$

$$= \frac{1}{7}x^7 + \frac{1}{35}x^{-5} + C \quad (C = C_1 + C_3)$$

⌈ Look at §6.1 31 to 42 and 54 to 72 ⌋

37: Compute $\int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx$.

$$\int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx =$$

$$\int \frac{1}{x}\, dx + \int \frac{1}{x^2}\, dx + \int \frac{1}{x^3}\, dx =$$

$$(\ln|x| + C_1) + \int x^{-2}\, dx + \int x^{-3}\, dx =$$

$$(\ln|x| + C_1) + \left(\frac{x^{-1}}{-1} + C_2\right) + \left(\frac{x^{-2}}{-2} + C_3\right) =$$

$\ln|x| - x^{-1} - \frac{1}{2}x^{-2} + C \quad (C = C_1 + C_2 + C_3)$

67: Compute $\int \left(x + \frac{1}{\sqrt{x}}\right) dx$.

$$\int \left(x + \frac{1}{\sqrt{x}}\right) dx = \int \left(x + (\sqrt{x})^{-1}\right) dx$$

$$= \int \left(x + x^{-\frac{1}{2}}\right) dx = \int x\,dx + \int x^{-\frac{1}{2}} dx$$

$$= \left(\frac{1}{2}x^2 + C_1\right) + \left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C_2\right)$$

$$= \frac{1}{2}x^2 + C_1 + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C_2$$

$$= \frac{1}{2}x^2 + C_1 + 2x^{\frac{1}{2}} + C_2 \quad (C = C_1 + C_2)$$

$$= \frac{1}{2}x^2 + 2x^{\frac{1}{2}} + C$$

85: Compute $\int x\sqrt{x}\,dx$.

$$\int x\sqrt{x}\,dx = \int x \cdot x^{\frac{1}{2}} dx = \int x^{\frac{3}{2}} dx$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + C = \frac{2}{5} x^{5/2} + C$$